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14. ABSTRACT

Turbomachinery rotors, or bladed disks, are known to suffer from severe vibration problems due to small, random deviations (mistuning) of the blade properties. Mistuning can lead to dramatic increases in the maximum blade stress and cause high cycle fatigue (HCF), which is a major cost, readiness, and safety concern for the U.S. Air Force. The primary objective of this research was to provide significantly improved understanding, modeling, and prediction of the vibration response of mistuned bladed disk systems by including the effects of important phenomena that had been largely neglected in previous mistuning models. The models developed in this research program were used to investigate the interaction of blade mistuning with aerodynamic coupling, stage-to-stage connections for multistage rotors, blade damage, and nonlinearities. In addition, key mistuning phenomena were examined through vibration testing of blisks (single-piece bladed disks). New methods were developed for identifying blade mistuning parameters from test data and for running experimental Monte Carlo assessments of the effects of mistuning on the system forced response.

15. SUBJECT TERMS

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NEXT-GENERATION MODELING, ANALYSIS, AND TESTING OF THE VIBRATION OF MISTUNED BLADED DISKS

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Abstract

Turbomachinery rotors, or bladed disks, are known to suffer from severe vibration problems due to small, random deviations (mistuning) of the blade properties. Mistuning can lead to dramatic increases in the maximum blade stress and cause high cycle fatigue (HCF), which is a major cost, readiness, and safety concern for the U.S. Air Force. The primary objective of this research was to provide significantly improved understanding, modeling, and prediction of the vibration response of mistuned bladed disk systems by including the effects of important phenomena that had been largely neglected in previous mistuning models. The models developed in this research program were used to investigate the interaction of blade mistuning with aerodynamic coupling, stage-to-stage connections for multistage rotors, blade damage, and nonlinearities. In addition, key mistuning phenomena were examined through vibration testing of blisks (single-piece bladed disks). New methods were developed for identifying blade mistuning parameters from test data and for running experimental Monte Carlo assessments of the effects of mistuning on the system forced response. To date, this work has been published in 13 conference papers [1–13], 3 journal papers [14–16], and parts or all of 5 doctoral dissertations [17–21].

1 Background

A bladed disk is typically designed to have identical blades. Such an idealized, "tuned" assumption allows one to use cyclic symmetry analysis to model the vibration of a bladed disk. An example tuned mode shape is shown in Fig. 1 for a finite element model of a bladed disk used in an industrial gas turbine. This mode features three nodal diameters.

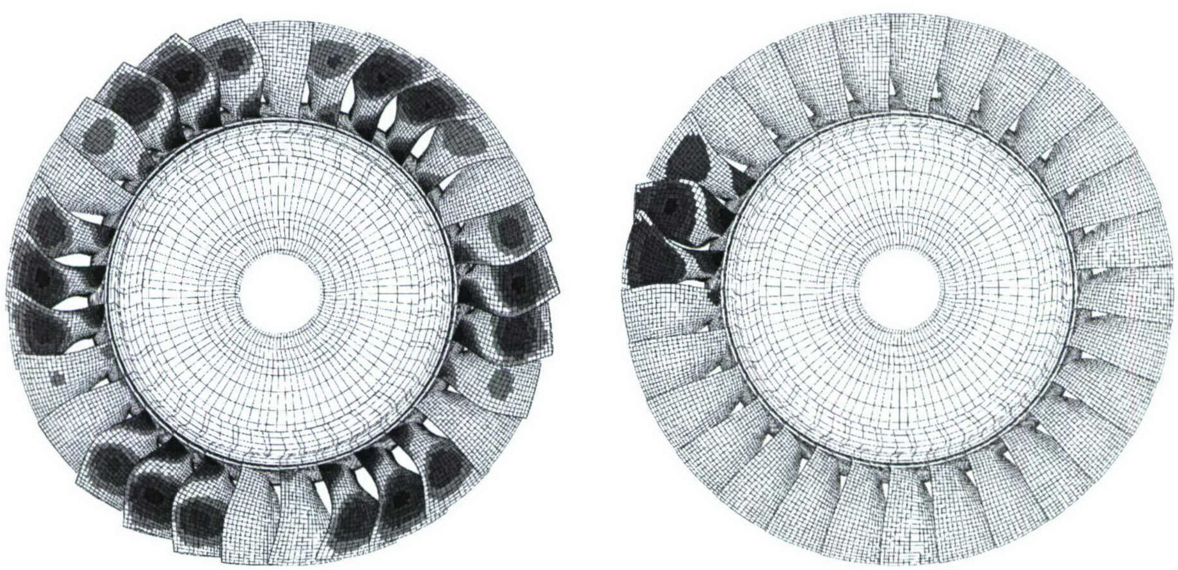


Fig. 1: Left: three-nodal-diameter mode shape of a tuned bladed disk. Right: localized mode shape of a mistuned bladed disk.

In reality, there are always random deviations among the blades due to manufacturing toler-

ances, wear, and other causes. This is called mistuning. Mistuning destroys the cyclic symmetry of the system. Therefore, a single sector model can no longer be used to predict the vibration of the full system. A mode shape for the industrial bladed disk with mistuning is shown in Fig. 1. The mistuned mode shape is not a pure nodal diameter mode, but instead it has multiple harmonic content, so it can be excited by all engine orders of excitation. Furthermore, the mode shape shows localization of the vibration about a few blades.

Even though mistuning is typically small (e.g., blade natural frequency differences on the order of a few percent of the nominal values), mistuned bladed disks can have drastically larger forced response levels than the ideal, tuned design [22–25]. The attendant increase in stresses can lead to premature high cycle fatigue (HCF) of the blades. HCF is a major cost, safety, and reliability issue for gas turbine engines. For example, in 1998, it was estimated by the U.S. Air Force that about 55 percent of fighter jet engine safety Class A mishaps (over $1 million in damage or loss of aircraft) and 30 percent of all jet engine maintenance costs were due to HCF [26]. It is clearly of great interest to be able to predict—and, ultimately, to reduce—the maximum blade response due to mistuning. The comprehensive modeling, analysis, and understanding of bladed disk vibration is thus critical to reducing the occurrence of HCF and improving the performance and reliability of turbine engines.

Bladed disk vibration first received significant attention from the research community in the late 1960s and the 1970s. Notable early work was done by Whitehead [22], Wagner [27], Dye and Henry [23], and Ewins [24, 28–30]. The bladed disk vibration literature has been surveyed in several review papers [31–34].

In order to capture the basic vibration characteristics, bladed disks have often been modeled as cyclic chains of spring-mass oscillators. The simplest such model of an N-sector bladed disk is a chain of N single-degree-of-freedom oscillators coupled by linear springs. Additional oscillators can be added at each sector to have both blade and disk degrees of freedom (DOF). The mistuning is typically modeled as small, random perturbations to the stiffnesses of the blade DOF. These lumped parameter models can be thought of as fundamental or qualitative models of mistuned bladed disks. For predicting the vibration response of an actual bladed disk used in a turbine engine, it is much better to take advantage of finite element models.

A finite element model is typically generated for only one sector of a bladed disk. Assuming that all the sectors are identical, cyclic symmetry routines can be used to calculate the free and forced response much more efficiently than modeling the entire system. However, not only does mistuning cause a possibly drastic change in the bladed disk dynamics, but it destroys the cyclic symmetry as well. Therefore, modeling just one sector is not sufficient; a full bladed disk model is needed. Modern industrial finite element models of a full bladed disk can be on the order of millions of degrees of freedom for a full bladed disk. Even with accelerated Monte Carlo simulation, using finite element analysis to predict the statistics of the mistuned forced response is not feasible. Therefore, reduction techniques are used to generate reduced-order models from a parent finite element model for a frequency range of interest.

The first generation of finite-element-based reduced-order models (ROMs) were based on component mode synthesis [35–37] (CMS) or similar component-mode-based techniques. In 1983, the application of CMS to reduced-order modeling of bladed disk vibration was investigated by Irretier [38]. In 1985, Zheng and Wang [39] used free-interface CMS to model groups of blades coupled through shrouds. Despite the promising findings of these initial investigations, it would be almost a decade before important new contributions were made in this research area.

In the 1990s, a new reduced-order modeling approach was developed by the investigators and co-workers [40–42], and it was found that a ROM on the order of $10N$ could provide good accuracy relative to the parent finite element model of an N-bladed disk. In this same time frame, Yang and Griffin [43] introduced a component-based reduced-order modeling technique that synthesized the disk and blade motion through an assumption of rigid blade base motion. These methods represented a leap in predictive capabilities, because they made it possible to generate reduced-order models systematically from finite element models, while generally retaining good accuracy and capturing the effects of mistuning. However, both approaches suffered from the coarse way in which the coupling at the disk-blade interface was captured. The disk-blade coupling is critically important because it relates directly to the interblade coupling, which has been shown to be a key factor for a bladed disk's sensitivity to mistuning [44].

Starting in 2001, more efficient reduced-order modeling methods [45–49] were developed. These methods could generate an ROM as small as N DOF for an N-bladed disk, typically by employing a basis of selected sets of tuned system modes. Furthermore, the ROMs generated with these methods retain high accuracy relative to the parent finite element model, but at a fraction of the cost. Thus, these techniques began what could be called a second generation of reduced-order modeling methods. Yang and Griffin [45] developed the subset of nominal modes (SNM) method in 2001. Feiner and Griffin [46] subsequently derived a simplified form of SNM for the case of an isolated family of blade-dominated modes, which they called a fundamental model of mistuning (FMM). Petrov *et al.* [47] proposed an alternative approach for efficient vibration modeling of mistuned bladed disks. In their formulation, the mistuned system forced response vector was expressed in terms of the tuned system forced response vector and a modification matrix.

The authors [48,49] developed a method that makes use of both tuned system modes and blade component modes to generate reduced-order models of mistuned bladed disks. A selected set of tuned system modes is used to form a basis for the ROM. However, for the purposes of modeling mistuning, the blade-alone motion is represented by a set of cantilevered blade modes and, optionally, the Craig-Bampton constraint modes [36] for the DOF that are held fixed in the cantilevered blade model. For this reason, it is called the component mode mistuning (CMM) method. Modal participation factors are calculated to relate the blade component modes to the blade motion in the tuned system modes, and this relation is used to project the individual blade mistuning onto the ROM. This mistuning projection approach is an extension of the method developed by Bladh *et al.* [41] for reduced-order modeling of mistuned bladed disks with shrouds. A notable feature of the CMM method is that it can handle various types of blade mistuning in a systematic manner, including non-uniform variations of individual blades that lead to different frequency mistuning patterns for different types of blade-alone modes.

However, even for these "second-generation" reduced-order models, there are still several simplifying assumptions that limit their capabilities for predicting the response of actual bladed disks in turbine engines. To illustrate this, consider Fig. 1, which shows the bladed disk as it is typically modeled for structural dynamic analyses: *in vacuo*, and isolated from other stages. In reality, a bladed disk is usually one stage of a multi-stage rotor, and it is subject to the effects of the fluid flow, which provides not only excitation but also damping and interblade coupling. Nonlinear phenomena associated with various physical and design aspects of turbine engine rotors provide additional complexities. Therefore, the overall goal of this research program was to provide significantly improved understanding, modeling, and prediction of the vibration response of mistuned bladed disk systems by including the effects of important phenomena that had been largely ne-

glected in previous mistuning models, thereby leading the way to "next-generation" methods.

2 Objectives

The main objectives of the research program were:

- To model and investigate the interaction of mistuning with other important physical aspects of bladed disks—such as aerodynamic coupling, stage-to-stage connections, blade damage, and contact nonlinearities—and to examine the influence of these mechanisms on the sensitivity to mistuning of the system response
- To examine the effects of various uncertainties—such as assumed boundary conditions and nominal modeling parameters—on simulation and system identification methods for mistuned systems
- To advance the state of the art in blisk vibration testing, with particular attention to the precise identification of blade mistuning and the careful study of the effects of mistuning and other uncertainties in a controlled experimental environment

It is believed that all of these objectives were met successfully by this research program, as described in the following sections.

3 Accomplishments and New Findings

This research program had several major accomplishments:

- Integration of aerodynamic coupling and damping terms into structural reduced-order mistuning models, and investigation of the combined effects of aerodynamic coupling and mistuning on the forced response
- Development of new reduced-order modeling and mode classification methods for mistuned multistage rotors
- Development of a reduced-order modeling method for the case of large, geometric mistuning (e.g., blade damage) or design changes
- Efficient modeling and nonlinear analysis of cracked blade vibration
- Investigation of the sensitivity of mistuning identification to measurement and modeling errors
- Development and experimental validation of a new mistuning identification technique with an integrated model updating procedure
- Introduction and validation of an experimental Monte Carlo mistuning assessment technique

These accomplishments are summarized in this section. The detailed mathematical formulations and results have been published in Refs. [1–21].

3.1 Aeroelastic Modeling of Mistuned Bladed Disks

Because the interblade coupling is so crucial to the mistuning sensitivity of a bladed disk, for some systems the inclusion of the coupling through the fluid might be critical to generating a meaningful model of the mistuned response. The vibration of mistuned bladed disks with both structural and aerodynamic coupling has been examined using relatively simple models in several previous studies [50–55]. However, recently developed reduced-order modeling methods provide a new opportunity to investigate this issue more thoroughly. Furthermore, the SNM [45], FMM [46], and CMM [48] methods all generate ROMs in tuned system modal coordinates, which allows easy incorporation of aerodynamic coupling. In fact, the CMM and SNM formulations were presented with unsteady aerodynamic terms included in the system equations of motion, and a version of the FMM method with aerodynamic terms has been introduced by Kielb *et al.* [56, 57].

Major advances were made in this research program by combining aerodynamic coupling with the CMM reduced-order modeling method to generate aeroelastic models of mistuned bladed disks [7–9, 14–16, 20]. In particular, aerodynamic effects have been included in a structural mistuning code to investigate the effects of aerodynamic coupling and damping on mistuned bladed disk response. Both aerodynamic influence coefficients and true aeroelastic system modes were calculated using a linear, unsteady, quasi-3D, frequency-domain aerodynamics code. Examples of the quasi-3D computational strips for a blade are shown in Fig. 2. It has been found that, for certain mode types, there are significant differences between the structure-only and aeroelastic mode shapes.

Furthermore, the influence of both aerodynamic coupling and aerodynamic damping can lead to notable differences in mistuned forced response results. The nonlinear dependence of aerodynamic forces upon vibration frequencies was accounted by using iterative calculations. A new hybrid technique was developed to study the iteration number needed to calculate the converged aerodynamic stiffness matrix. In order to predict the iteration number efficiently, the nonlinear dependence of the unsteady aerodynamic force upon the system frequency was approximated using a linear correlation. The prediction results were validated with the actual simulation results. The effects of some system parameters on the convergence history of aeroelastic calculations were studied using the new hybrid technique. It was found that the magnitude of generalized aerodynamic forces and the gradient of generalized aerodynamic forces with respect to the vibration frequency are the two most important factors. In a numerical investigation, the aerodynamic effects were found to have a significant influence on mistuned forced response level, which is seen in Fig. 3. Note that there is a very large difference between results calculated with and without aerodynamic terms. For the results with aerodynamic effects included, there is also a difference between results for which the aerodynamic stiffness matrix was calculated directly using the tuned system modes versus using the cantilevered blade normal modes ("Converged" versus "Blade"). Also note the difference between the one-step results and the converged results from the iterative calculations mentioned above ("One-Step" versus "Converged").

Fig. 2: Quasi-3D computational grids used for the aerodynamic analysis.

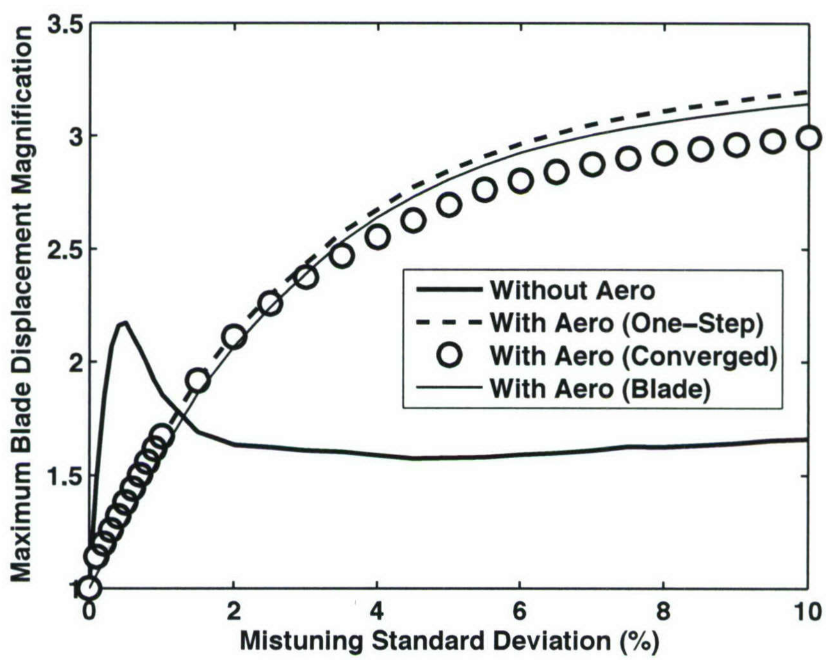


Fig. 3: Effect of aerodynamic coupling and damping on the 95th percentile mistuned vibration response.

3.2 Reduced-Order Modeling and Mode Classification of Multistage Rotors

Bladed disks are typically modeled as isolated systems, but in general they are actually connected to adjacent stages in a multi-stage rotor. Prior to this research program, a study by Bladh *et al.* [58] showed that connecting a second stage to a single-stage finite element model of a bladed disk could lead to significant changes for predictions of the maximum blade response. For some operating conditions, dramatic changes in the first stage's sensitivity to mistuning were observed. This was explained by the presence of the adjacent stage, which alters blade-to-blade coupling through the disk and thus mistuning sensitivity. Furthermore, it was found that applying constraints to the boundary degrees of freedom of a single stage could not faithfully capture the boundary conditions of the actual stage-to-stage connection.

In this research program, new reduced-order modeling and mode classification methods were developed for mistuned multistage rotors [3,4,12,18]. The main challenge in modeling multistage rotors is that the stages have different numbers of blades, and thus the single-sector finite element models for adjacent stages have a mismatch in both sector sizes and meshes. Therefore, a stage-by-stage component mode synthesis approach was adopted, with each stage being treated as a separate component. Thus, cyclic symmetry analysis can be used to calculate component normal modes and constraint modes for each stage. Furthermore, the constraint modes are transformed to a common basis of harmonic interface shapes in order to enforce displacement compatibility between stages, despite the fact that the finite element node locations do not match.

Some free vibration response results for an example two-stage system are shown in Fig. 4. On the left, a mode shape that is confined to stage 2 is shown. On the right, a mode shape that is coupled across stages is shown. Clearly, this type of mode shape could not be captured with a single-stage model. It was found that multistage coupling and the stage-to-stage periodicity mismatch can have a large effect on the forced response as well for certain operating conditions that excite these coupled modes.

In order to better predict and understand these multistage modes, a multistage mode classification method was developed [12,18]. This method uses information generated during the reduced-order modeling process to efficiently estimate the amount of strain energy contained in each stage for every multistage mode, without having to project back to finite element coordinates. This allows one to automatically generate natural frequency versus nodal diameter plots for each stage. It also allows one to identify which modes are coupled across stages. The plot for stage 2 of the example two-stage rotor is shown in the center of Fig. 4. It has been found that many multistage system modes can be associated with single-stage nodal diameter modes, as seen in this natural frequency plot. However, in disk-blade mode veering regions, coupled modes are also found. These coupled modes (marked as circles) tend to deviate from the true single-stage modes (shown as lines) on the frequency plots.

In Fig. 5, a numerical validation of the reduced-order modeling method is presented. The forced response results from the finite element model (FEM) and a reduced-order model (ROM) of the two-stage rotor are compared for engine order excitation on one stage. Both tuned and mistuned cases are shown. It is seen that the ROM results are very close to those of the full FEM, and that the effects of mistuning on the increase in forced response are well captured.

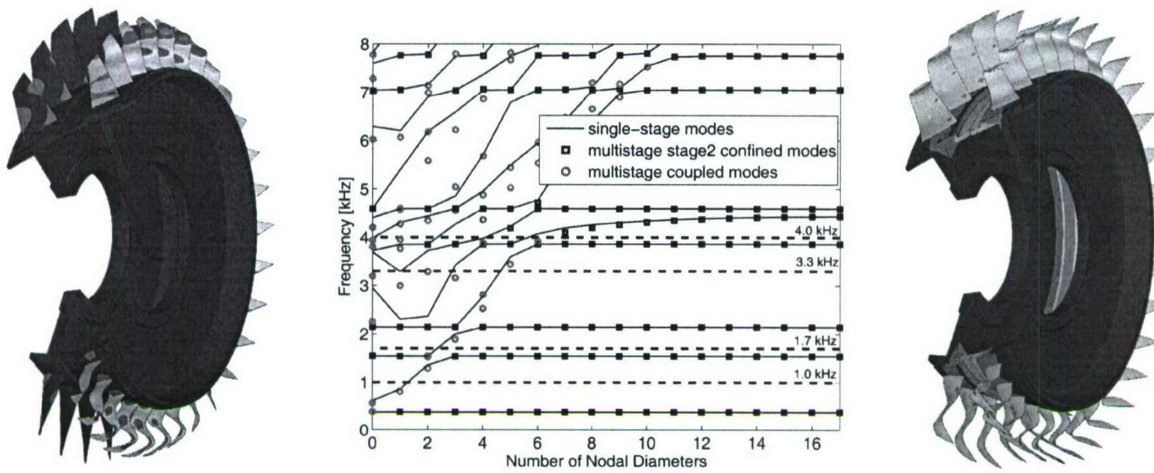


Fig. 4: Left: a multistage mode shape that is mostly confined to stage 2. Center: Multistage mode classification results for stage 2. Right: a multistage mode that is strongly coupled across stages.

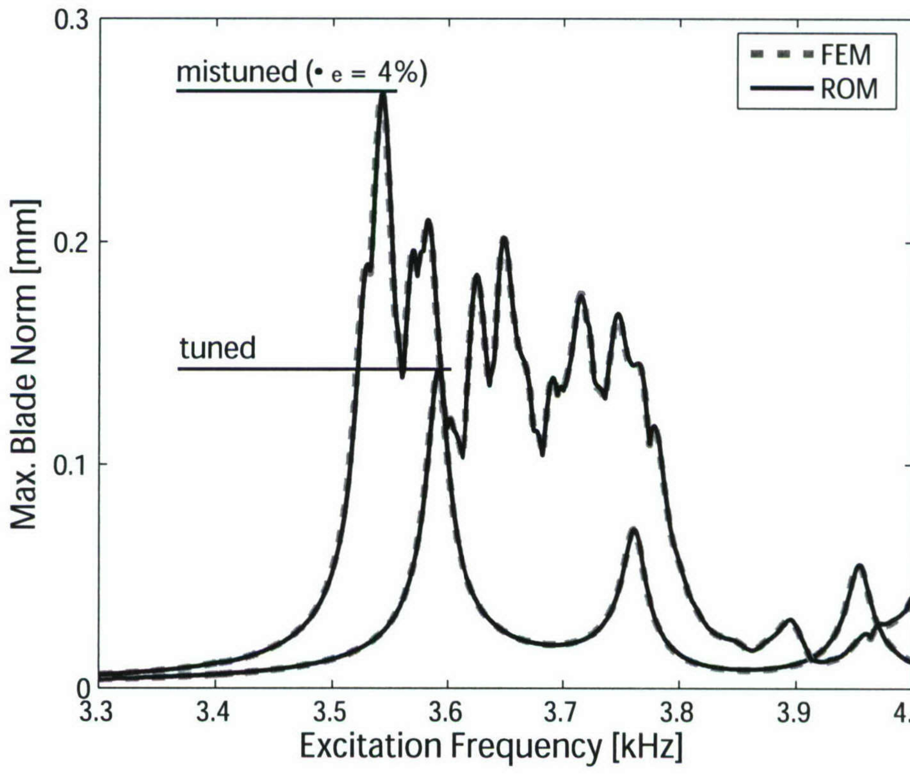


Fig. 5: Mistuned and tuned forced response results for the two-stage rotor subject to engine order excitation show an excellent match between the full FEM and the ROM.

3.3 Modeling of Blade Damage and Geometric Design Changes

In addition to small mistuning, the effects of large, geometric changes due to blade damage (e.g., a dent due to a bird strike, or missing material in the blade tip due to fracture) were investigated. Such geometric changes to the blade result in blade-mode-shape mistuning. Various finite-element-based ROMs have been proposed in the literature for bladed disks with small mistuning, such as the CMM method that was developed by the investigators. Many of these techniques rely on the fact that mistuned-system normal modes can be effectively represented using a linear combination of the normal modes of the nominal (tuned) system. However, when the mistuning or geometric deviation is large, the number of tuned-system normal modes that are required to describe the mistuned-system normal modes increases dramatically. This makes it impractical to use these previously developed ROMs for modeling the effects of severe damage.

To address this, a new reduced-order modeling approach for bladed disks subject to damage or geometric design changes was developed in this research program [1, 17]. This new technique employs a mode-acceleration formulation with static mode compensation (SMC) to account for the effects of the geometric changes on the mode shapes. Therefore, it is referred to as the SMC method here.

By accounting for the effects of mistuning as though produced by external forces, a set of basis vectors can be established using a combination of tuned-system normal modes compensated by static modes. The obtained basis vectors approximately span the space of the mistuned-system modes without requiring a much more expensive modal analysis of the mistuned system, and they provide much better convergence than tuned-system normal modes. Furthermore, in order to extend the method to higher frequency ranges, quasi-static modes, in which inertia effects are included, are employed in place of static modes in the mode-acceleration formulation. ROMs based on the new SMC technique are extremely compact, yet they accurately capture the vibration response of bladed disks subject to geometric mistuning or design changes.

Several example cases were examined [1, 17], including a system featuring one blade with significant geometric mistuning due to foreign object damage, a system with a fractured blade, and a system subject to geometric design changes in the disk. In Fig. 6, the FEMs of a blade with two levels of foreign object damage are shown on the left, and a mode shape localized about the damaged blade is shown on the right. In Fig. 7, a blade-alone mode shape is shown for a fractured blade. Also shown in Fig. 7 are the forced response simulation results for the full bladed disk with one fractured blade. Note that the results from a 36-DOF ROM based on the SMC method match extremely well with those from a 126,846-DOF FEM. Also note that the fractured blade introduces a new resonance in the system forced response that is not present for the original, tuned system (system with no damage).

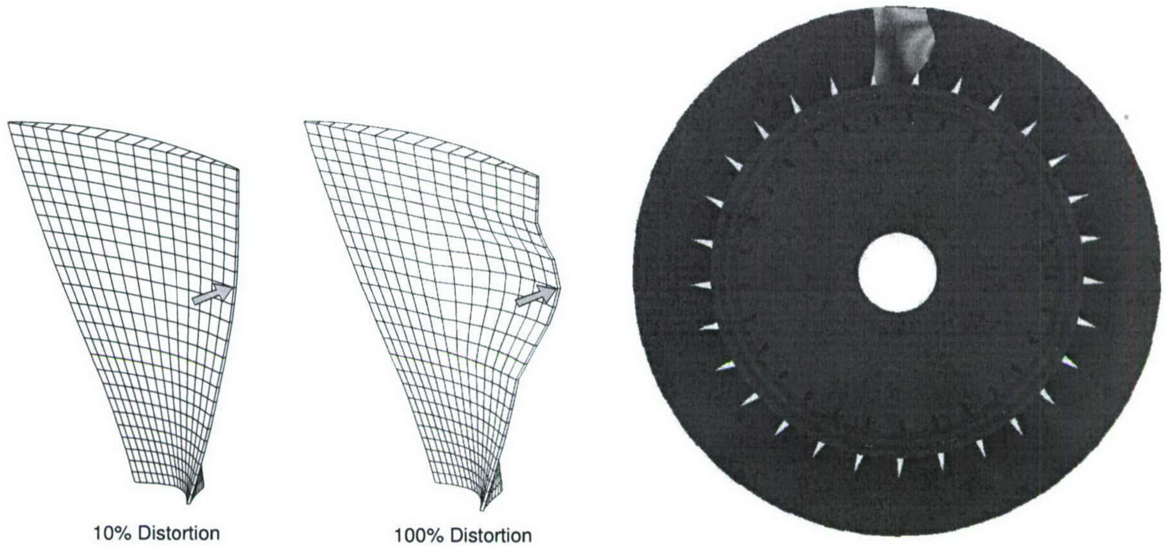


Fig. 6: Left: two levels of distortion in a model of a blade with foreign object damage. Right: a mode shape of the blisk shows localization about the damaged blade.

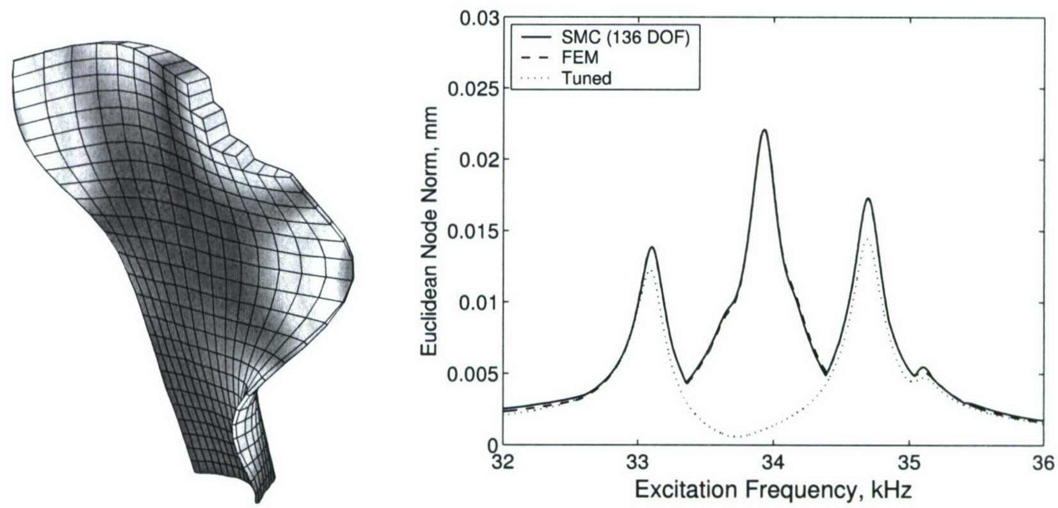


Fig. 7: Left: blade-alone mode shape of a fractured blade. Right: forced response for the tuned system and the damaged system (full blisk with one fractured blade) as calculated with the finite element model and the ROM generated with the SMC method.

3.4 Nonlinear Vibration Analysis of Cracked Blades

There are many examples of important nonlinear phenomena in bladed disks: contact at shroud interfaces, at dovetail attachments for inserted blades [59, 60], and due to rubbing between blade tips and the engine casing [61]. There are also nonlinearities introduced by vibration reduction elements such as impact dampers [62, 63] and dry friction dampers. Another important example of nonlinearity in bladed disks is the vibration of cracked blades. This is a piecewise linear system due to the opening and closing of the crack during each vibration cycle.

In this research program, a new modeling approach and efficient nonlinear vibration analysis method was developed for cracked blades [6, 11, 13, 21]. A reduced-order model was generated with a component mode synthesis approach by selecting a small set of linear modes plus retained physical degrees of freedom (constraint modes) for the crack surfaces. The nonlinear equations due to intermittent contact at the crack surface were solved efficiently using a hybrid frequency/time-domain (HFT) approach.

A finite element model of a cantilevered cracked beam subject to harmonic excitation was used for validation purposes. The nonlinear, steady state vibration response of the cracked beam was calculated using both the time integration method and the HFT method, and the results were compared. The finite element model is shown on the left in Fig. 8. The finite element model had 9780 DOF. The reduced-order model had 62 DOF, which includes 20 normal modes and 42 "active" DOF. Of the active DOF, 1 DOF was kept for each of the 40 nodes on the crack surfaces so that the relative motion of the nodes could be tracked during each iteration of the HFT method. In addition, 2 active DOF were kept for nodes at the tip of the beam in order to apply external forcing and to track the tip motion.

The frequency response curves obtained by the time integration method and the HFT method are shown on the right in Fig. 8. The time integration was performed using the commercial code ANSYS. The Newmark method was used for the time integration, and the Augmented Lagrangian method was used as the contact algorithm. The HFT method was applied with several values of parameters. As can be seen, the results calculated by the HFT method show an excellent agreement with the results by the time integration method. Furthermore, the HFT method was two to four orders of magnitude faster than the time integration method in terms of computational time.

In addition, the modeling procedure was extended to account for the effects of rotation on the forced response [11]. Resonant response results for a representative cracked blade model rotating at 5000 RPM are shown in Fig. 9. It was found that the nonlinear, rotating system exhibited the jump phenomenon at some forcing levels. The jump phenomenon is seen as apparent discontinuities plot for the lower forcing levels shown on the right in Fig. 9.

Recent work has examined the interaction of a cracked blade and mistuning on the system response of a bladed disk [13]. Future work in this area will have important applications to damage detection, structural health monitoring, and prognosis for rotors in jet engines and other turbomachinery.

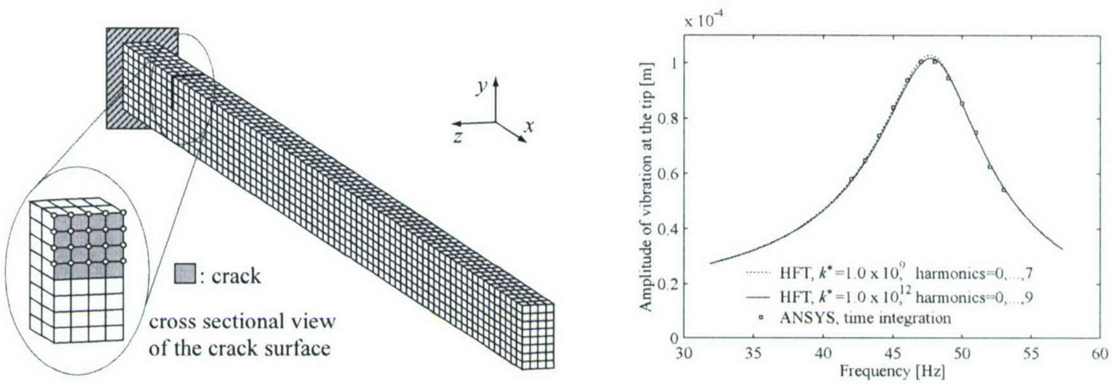


Fig. 8: Finite element model of the beam and the validation results.

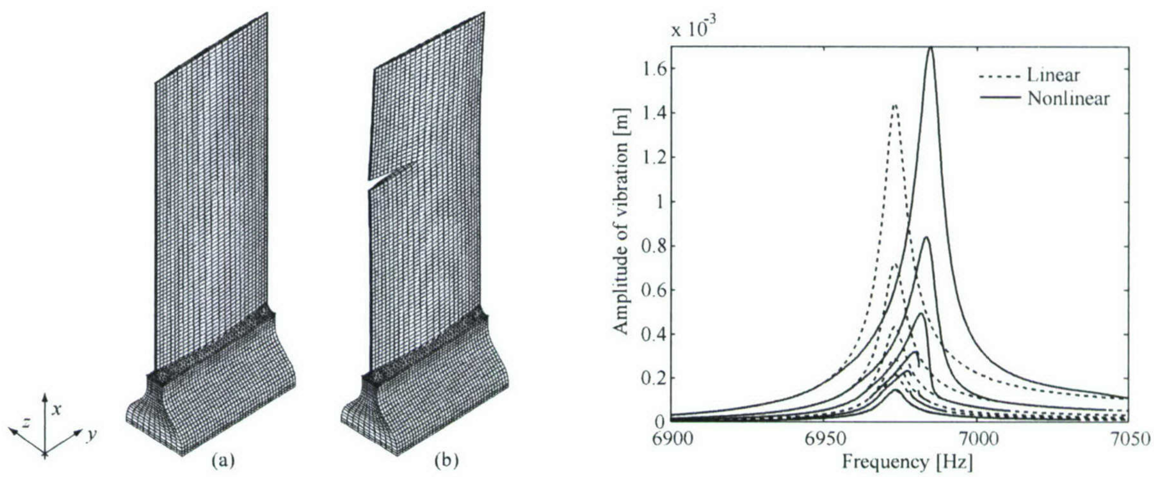


Fig. 9: Left: FE model of the cracked blade without (a) and with (b) rotation. Right: Forced response results for a cracked blade model rotating at 5000 RPM.

3.5 New Mistuning Identification Technique with Model Updating

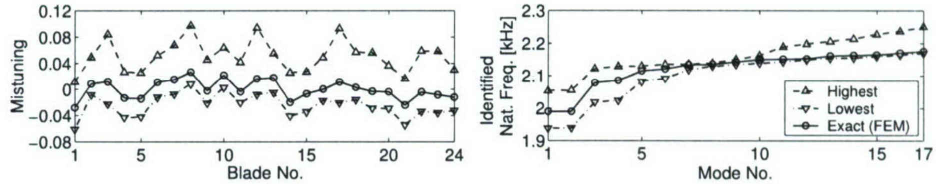
An important practical consideration for bladed disk vibration research is how to identify the mistuning that is actually present in a manufactured bladed disk. For rotor stages with inserted blades, the blade-alone natural frequencies can be measured directly to determine mistuning values to be used in simulations as well as to estimate mistuned blade structural dynamic properties [64–66]. However, for a blisk—a one-piece bladed disk—the blades cannot be removed from the assembly. Therefore, mistuning identification techniques based on experimental measurements of system response have been developed recently to determine the individual blade mistuning pattern for each blade-dominated mode family of interest. Mistuning identification can also be used to assess the quality of the manufacturing process, to flag possible tooling problems, and to perform structural health monitoring of turbine engine rotors.

The first such mistuning identification technique was developed by the authors in a previous AFOSR research project [67–69]. In order to identify individual blade mistuning from the vibration response of an entire bladed disk, two sources of information were used: a finite element model of the bladed disk, and a set a measurements of the response of the actual bladed disk. Either mode shape measurements or forced response measurements can be used [69]. A similar technique has recently been developed by Feiner and Griffin [70, 71].

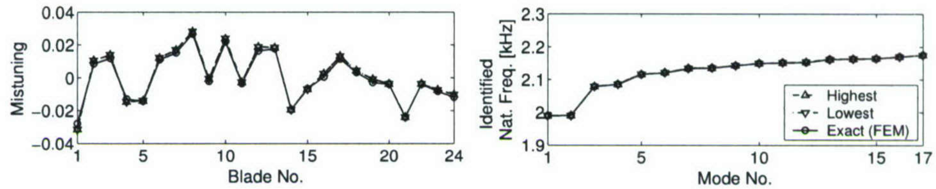
In this research program, a new mistuning identification method was developed [2, 17] based on the CMM method. This new technique also incorporated a reduced-order model updating procedure. This enhancement was motivated by the fact that mistuning is not the only form of uncertainty in a turbomachinery rotor. For example, modeling assumptions and limitations could introduce significant uncertainty for forced response predictions. Moreover, even when some aspect of the dynamics is well modeled, certain parameter values may not be precisely known, which is another form of uncertainty. Therefore, the sensitivity of mistuning identification results to uncertainties in the measurements and models used in the identification process was examined. A numerical study was carried out for a blisk with 24 blades (see Fig. 15). Random variations (sampled from a uniform distribution with a range of +/- 5% relative to the nominal value) were assigned to various system parameters and measured data used in the mistuning identification process in order to determine their influence on the calculated mistuning values. As seen in Fig. 10, the identified blade mistuning values were very sensitive to errors in the tuned-system eigenvalues. Errors in the tuned-system eigenvalues are the difference between the natural frequencies of the tuned finite element model and those of the virtual tuned system that may be inferred from measurements of the actual bladed disk. Here, this difference is referred to as the "cyclic modeling error", because the deviation of the parent tuned FEM from the virtual tuned system features cyclic symmetry.

Based on the numerical study described above, the mistuning identification technique was then integrated with a model updating method in order to improve the robustness of the method. In particular, the equations of motion were modified to include a cyclic modeling error term so that, in addition to the blade mistuning values, the cyclic modeling error values could be identified.

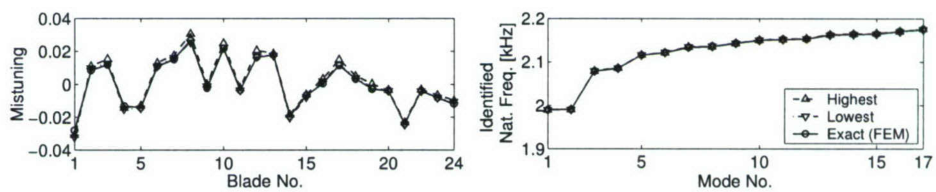
In this manner, the CMM-based mistuning identification technique was enhanced. Figure 11 shows forced response predictions from a CMM reduced-order model using mistuning values identified with the original (without model updating) and enhanced (with model updating) mistuning identification techniques. It can be seen that the enhanced technique with updating is more robust and leads to more accurate results.



(a) With 5% random errors in tuned system natural frequencies



(b) With 5% random errors in structural damping



(c) With 5% random errors in measured response

Fig. 10: Sensitivity of identification results to errors in model parameters and measured data.

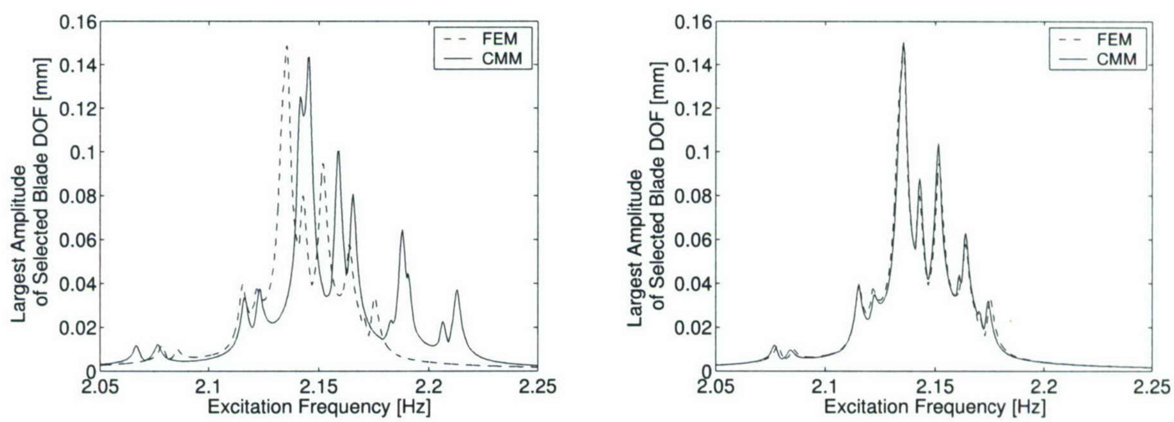


Fig. 11: Accuracy of forced response predictions (relative to finite element results) from the CMM model using the identified mistuning parameters. Left: using parameters from the original mistuning identification method. Right: using parameters from the enhanced mistuning identification method with model updating.

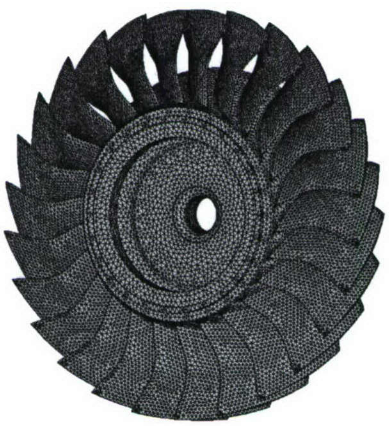
3.6 Experimental Validation of the Mistuning Identification Technique

The new mistuning identification method was experimentally validated in this research program [5, 19]. One of the test specimens used to validate the method was an advanced NASA blisk prototype with 26 blades, which is shown in Figure 12. A special fixture was designed and manufactured to mount both the blisk and speakers on the vibration-isolated table.

The finite element model (FEM) of the blisk is also depicted in Fig. 12. The CMM technique was used to generate a reduced-order model with 26 DOF, where the 26 modes belonged to the first blade-dominated mode family (first flexural bending modes of the blade).

Figure 13 shows the values of blade mistuning and cyclic modeling error that were identified using the new, enhanced mistuning identification technique. The identified parameters were then used to generate a mistuned reduced-order model of the blisk, and the modes of the mistuned blisk were predicted using this ROM. For validation, the mode shapes were measured experimentally.

A comparison between the predicted mode shapes (based on mistuning identification results) and measured modes shapes is shown inf Fig. 3.6. Figure 14(a) shows good agreement between the predicted and measured mode shapes at 738.9 Hz, while Fig. 14(b) shows a nearly perfect match at 764.1 Hz. Figure 14(b) also shows severe localization of the mode shape around blades 18 and 19, indicating the high sensitivity to mistuning in the specimen tested. Note how well the updated CMM model is able to capture this localized behavior.



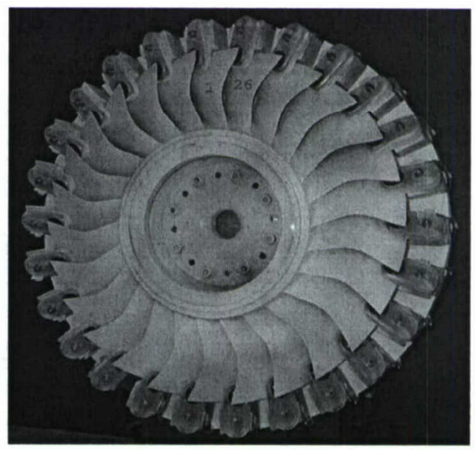
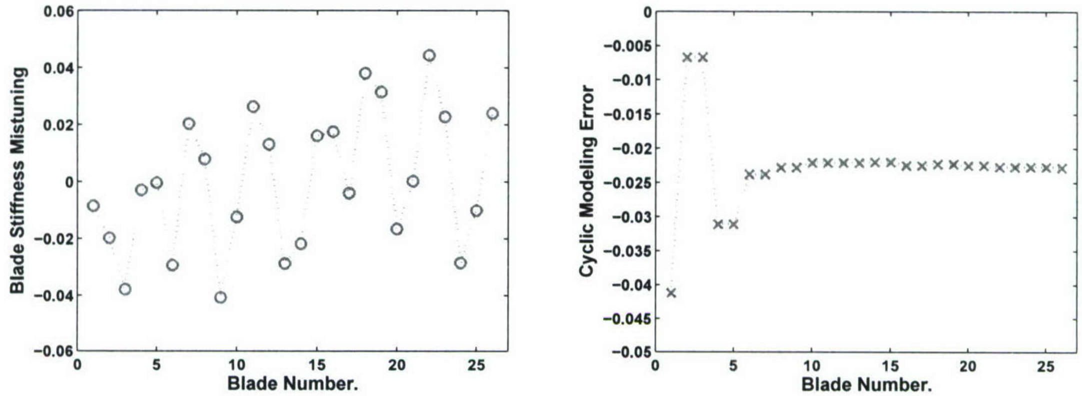


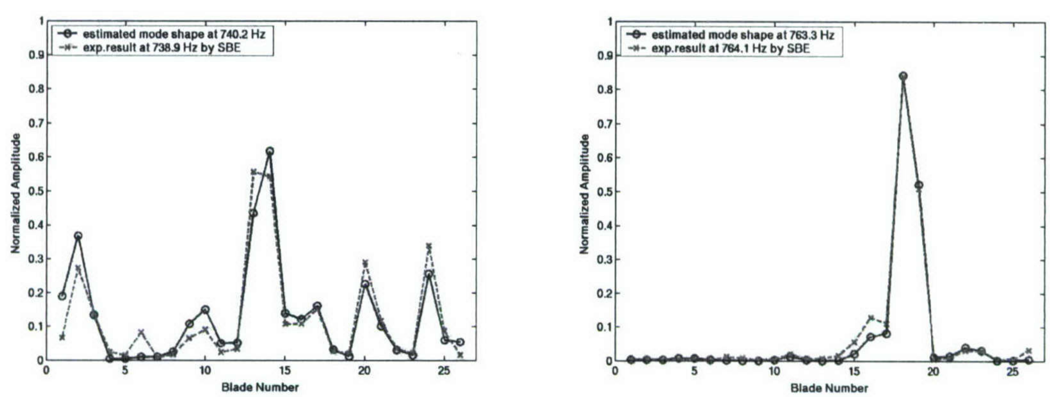
Fig. 12: The finite element model and tested prototype of the NASA blisk.



(a) Identified blade mistuning

(b) Identified cyclic modeling error

Fig. 13: Identified parameters of the blisk based on the experimental results.



(a) 740.2 Hz

(b) 763.3 Hz

Fig. 14: Mode shapes predicted by a CMM model with identified mistuning parameters, compared to the mode shapes measured by a laser vibrometer while subjecting the blisk to single blade excitation (SBE).

3.7 Experimental Monte Carlo Mistuning Assessment Method

In order to estimate the statistics of the forced response for a population of randomly mistuned bladed disks with the same nominal design, a Monte Carlo simulation may be performed. First, given a value for the standard deviation of random mistuning, the mistuned blade stiffnesses for one realization of a mistuned bladed disk are assigned by a pseudo-random number generator. Second, a frequency sweep is performed to find the largest peak response amplitude of any blade on the bladed disk. Third, this process is repeated for many realizations of mistuned rotors.

Depending on the mistuning strength and the sensitivity of the system, it may require many thousands or even tens of thousands of realizations to estimate the probability density function of the worst-case blade response, especially for capturing the tails of the distribution (e.g., the 99th percentile). However, it is important to note that the variable of interest is the largest forced response amplitude found for any blade in a frequency sweep. Therefore, the theory of extreme value statistics [72, 73] can be applied to the problem. A remarkable result from this area of probability theory is that the distribution of the maximum of a set of independently and identically distributed random trials approaches one of three extreme value distributions as the number of trials becomes large. It was shown by Castanier and Pierre [74] that the distribution of the largest-responding blade amplitudes will asymptotically approach the third extreme value distribution, which is the Weibull distribution. This conclusion has also been confirmed by Mignolet *et al.* [75]. Therefore, the statistics of the mistuned forced response can be estimated by fitting the Monte Carlo results from relatively few realizations (e.g., 50 realizations) to the Weibull distribution, which reduces computational costs by orders of magnitude. This accelerated Monte Carlo simulation procedure was presented by Castanier and Pierre [74] and by Bladh *et al.* [42].

However, in order to examine various mistuning patterns in a test environment, the blades would have to be physically changed (e.g., by attaching masses), which is time-consuming and cumbersome. Therefore, an experimental Monte Carlo mistuning assessment technique has been developed and validated in this research program [10]. This novel testing procedure is analogous to a computational Monte Carlo simulation. The investigators discovered that any structural mistuning pattern has an equivalent forcing mistuning pattern in the equations of motion. That is, theoretically, mistuning could be implemented in the forcing function to mimic structural changes and achieve the same mistuned forced response. This was validated experimentally using a 24-blade rotor. In Fig. 15, the probability density function (PDF) and cumulative distribution function (CDF) estimated from a Monte Carlo simulation of 80 randomly mistuned rotors with standard deviation of mistuning 0.1% is compared to that estimated from the experimental assessment of the same 80 mistuning patterns. It is seen that the agreement between numerical and experimental results is outstanding, especially for high-percentile response.

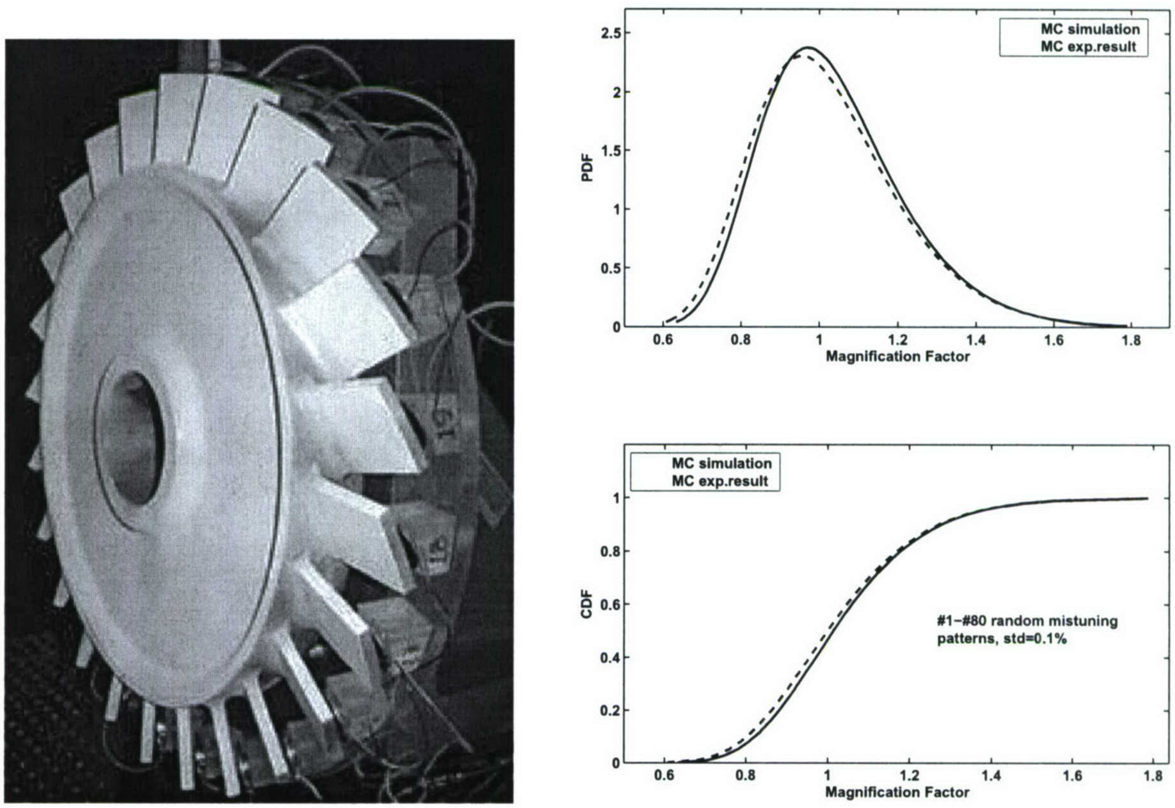


Fig. 15: Left: the "validation blisk" used for the experimental study. Right: PDF and CDF for the mistuned forced response of the validation blisk from a computational Monte Carlo simulation and an experimental Monte Carlo mistuning assessment.

4 Publications

To date, this work has been published in 13 conference papers [1–13], 3 journal papers [14–16], and 5 doctoral dissertations [17–21]. These publications are listed as the first 21 entries in the References section below. Several other papers have been submitted to archival journals or are being prepared for journal submission.

5 Interactions and Transitions

During the first two years of this research program, the progress achieved in this program was communicated to the Air Force Research Laboratory (AFRL), NASA, and several turbine engine companies through meetings of the GUIde Consortium on blade durability. For example, the method for integrating aerodynamic terms into mistuning models (section 3.1) was presented, because it provided important new capabilities for aeroelastic modeling and simulation of turbomachinery rotors that were of interest to the consortium members.

In addition, new projects have been initiated with industry and leveraged to further develop methods and transfer technology from this AFOSR grant. In particular, the multistage (section 3.2) and cracked blade (section 3.4) modeling methods are being further developed with Pratt & Whitney through a subcontract on a DARPA Engine System Prognosis project. Also, the mistuning identification (section 3.6) technology will be further developed with GE Aviation through their University Strategic Alliance program.

Finally, the principal investigator is participating in a new SBIR (Small Business Innovative Research) project sponsored by AFRL. An important goal of the project is to combine the new nonlinear vibration analysis method for cracked blades (section 3.4) with an advanced fracture analysis code to develop improved fatigue life prediction capabilities for turbomachinery rotors.

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